PHILOSOPHICAL TRANSACTIONS.

I. *The greatest Effect of Engines with uniformly accelerated Motions considered.* By Francis Blake, *Esq; F. R. S.* *

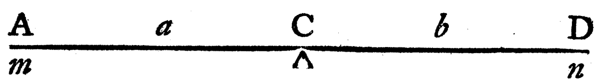
Read June 24, 1756.

THE writers, I have met with, upon the *maximum* of Engines, or the greatest effect possible in any given time, have supposed the working parts of the machine to retain their direction, and be uniformly moved by the force of a current. They have therefore considered only the case of an uniform Rotation, as in the action of grinding; where the impediments and impulses being brought to a balance, the impulses are but sufficient to prevent a decay in the generated motion. And, upon that view of the Problem, the load of an Engine, when the

* The distance of time betwixt the reading of this paper, and its now appearing in the *Transactions*, was owing to the absence of the author, and his desire to reconsider it before it went to the press.

the effect is a maximum, and the force a current, is determined by computation to be *four ninths* of the weight which would cauſe the Engine to reſt. This, then, being ſuited only to an uniform velocity both in the lever and obſtacle, I would conſider the caſe of an uniformly accelerated one in repeated vibrations. The maximum which correſponds to it is adapted to the ſteam-engine, and of no leſs importance to be determined than the other. And, no doubt, theſe ſpeculative inquiries into mechanical ſubjects, were they even barely conſidered as our guides in experiment, may appear to deſerve the attention of the Society. But indeed, and I ingenuouſly confeſs it, an opportunity to make ſome remarks upon what I formerly laid before you, concerning the proportion of the cylinders, was a further inducement to the preſent reſearch.

A general expreſſion for the time of a ſtroke in ſuch vibratory Engines, will lead us without trouble to a computation of their effects.



Let A D be a lever, whoſe Brachia are a and b, and ſuppoſed without weight. Let m be a Power, and n a weight. Then $a : b :: n : \frac{bn}{a}$, the balance for n at A, and $m - \frac{bn}{a}$ is the effective force at A, which multiplied by the lever a gives $ma - nb$ for the efficacioufneſs of that force in the angular velocity of the Power and weight. Now, by the principles of Mechanics, the Inertia of any bodies revolving

volving about a Center is as the quantities of matter into the ſquares of the Brachia; and in the preſent caſe, therefore, the whole Inertia of m and n is as $ma^2 + nb^2$. Hence then, and becauſe the velocity generated in a given particle of time is as the Force directly and Inertia inverſely, we have $\frac{ma - nb}{ma^2 + nb^2}$ as the accelerating force, or the meaſure of the angular velocity of the Power and weight at the end of the ſaid given particle of time. And I uſe the angular velocity, becauſe the arbitrary proportions in the lengths of the Brachia which may form an Equilibrium will not alter the expreſſion. But again, the times of deſcent by means of uniform forces, thro' a given ſpace, are inverſely as the ſquare roots of the accelerating forces, or meaſures of the velocities generated in a given particle of time; and therefore $\sqrt{\frac{ma^2 + nb^2}{ma - nb}}$ is a general expreſſion for the time of a ſtroke. This being had, the ſolution is eaſy; for, ſuppoſing n only to be variable, ſay as $\sqrt{\frac{ma^2 + nb^2}{ma - nb}}$ $: n ::$ 1, a conſtant or given time: $n\sqrt{\frac{ma - nb}{ma^2 + nb^2}}$ the effect in time 1, *ex hypoth.* the greateſt effect which can poſſibly be produced in the ſaid given time. Taking, then, as uſual, the Fluxion equal 0, we have, after a proper reduction, $2a^3m^2 - 3a^2mnb + amnb^2 - 2n^2b^3 = 0$, and $n = \frac{am}{b}\sqrt{\frac{a}{b} + \left|\frac{3a - b}{4b}\right|^2} - \frac{am \times 3a - b}{4b^2}$. Therefore, in theſe ſorts of Engines, when the Brachia are given, the weight :

 Power

Power :: $\frac{a}{b}\sqrt{\left(\frac{a}{b}+\frac{3a-b}{4b}\right)^2-\frac{a\times\overline{3a-b}}{4b^2}}$: 1; and if the Brachia are equal, *i. e.* if $a=b$, the weight : power :: $\sqrt{\frac{5}{4}}-\frac{1}{2}$: 1, *viz.* 0,618 : 1 nearly when the effect is a maximum. And ſo, in like manner, when b, m and n are given, and a is made variable, it is eaſy to ſee that, inſtead of the load, the beſt diſtance of the power from the fulcrum of the lever will be the reſult of the proceſs; *viz.* $a : b :: n + \sqrt{n^2+mn} : m$. But, this by the way.

In the proportion here determined, the power m is a weight, and therefore $ma-nb$, which is the generating force, being partly employed to overcome the Inertia of the quantity of matter m, it is not wholly taken up in giving motion to the weight n; and the relative velocity is continually decreaſing. But, on the other hand, if m be the force of a ſpring, as is that of our atmoſphere, or if n can be uniformly accelerated any how, in repeated vibrations, that there may be no ſenſible diminution of the relative velocity, the whole will be exerted on the weight to be raiſed; *i. e.* the tenſion of the rope or chain, by which the power is confined to act on the weight, will always be the ſame as tho' the Beam were at reſt; and then, by expunging ma^2 out of the expreſſion for the greateſt effect, $n\sqrt{\frac{ma-nb}{ma^2+nb^2}}$ becomes evidently enlarged to $n\sqrt{\frac{ma-nb}{nb^2}}$. The conſequences are theſe. 1ſt, The greateſt effect of this engine when m is a ſpring, will always exceed the cotemporary effect where m is a weight. 2dly, The proportion of the Power and weight will then be be $n : m :: a : 2b$, as appears by taking the fluxion of $n\sqrt{\frac{ma - nb}{nb^2}} = 0$, and reducing the equation in the manner above. Whence the load to be raiſed for the greateſt effect of a ſteam-engine, if the Inertia of the materials compoſing its working parts be put out of the queſtion, will be juſt half of what is ſufficient to balance the atmoſphere, whether the Brachia of the lever be equal or not.

Permit me now to trouble you with two or three remarks on what I formerly laid before you concerning the proportion of the cylinders. And, 1ſt, in all values of the Brachia, with regard to their lengths, and all values of n, the expreſſion $\sqrt{\frac{ma^2 + nb^2}{ma - nb}}$ for the time of a ſtroke, when m is a weight, is the general expreſſion to be uſed for the time. 2dly, m being conſidered as a ſpring, the time of a ſtroke is as $\sqrt{\frac{nb^2}{ma - nb}}$; and then if, according to what I have there directed, a be taken variable, and m the reciprocal of a, the advantages to be gained by the breadth of the cylinder can only ariſe from a diminution of friction, and from the matter in the Beam; for, the expreſſion $\sqrt{\frac{nb^2}{ma - nb}}$ becomes conſtant, and thence the ſtrokes are iſochronal. I might, furthermore, proceed to examine into theſe advantages, more explicitly than is there done, upon the principles laid down, when m is a weight. But many particulars (ſuch as the form of the Brachia and various appendages, with their quantities of matter and centers of gyration)

gyration) being wanting to perfect the Theory of the construction, I shall drop the inquiry when I have made only one remark more. It is this. The shortness of the Brachia diminishes the resistance of the Engine to motion; and, therefore, the inequality which I proposed in them was in part to avail myself of that obvious advantage, without incurring the inconvenience of enlarging the pump-bores. I say it is an obvious advantage; for, the matter in the Brachia, that the Equilibrium may be preserved, being inversely as their lengths, and the resistance to motion in the direct ratio of the squares of those lengths, the resistance of the longer arm is to that of the shorter as the lengths of them directly.

Queen-square, Westminster,
June 22, 1756.

II. *Observa-*